Wronskian

If $y_1, y_2, \ldots, y_n$ be n sol^n of nth order dif eq^n then by the Wronskian of these sol^ns we meant $W(y_1, y_2, \ldots, y_n)$ and is defined as

$$W(y_1, y_2, \ldots, y_n) = \begin{vmatrix} y_1 & y_2 & \cdots & y_n \\ y_1' & y_2' & \cdots & y_n' \\ y_1'' & y_2'' & \cdots & y_n'' \\ \vdots & \vdots & & \vdots \\ y_1^{n-1} & y_2^{n-1} & \cdots & y_n^{n-1} \end{vmatrix}$$

The set of sol^n $y_1, y_2, \ldots, y_n$ are linearly independent if $W(y_1, y_2, \ldots, y_n) \neq 0, \forall x$.

and linearly dependent if $W(y_1, y_2, \ldots y_n) = 0, \forall x$

Ex. Find the wronskian of $\{\sin 2x, \cos 2x\}$ and interpret the result.

Ans: We have $W(\sin 2x, \cos 2x)$

$$= \begin{vmatrix} \sin 2x & \cos 2x \\ 2\cos 2x & -2\sin 2x \end{vmatrix}$$

$$= -2 \neq 0 \quad \forall x$$

$\therefore$ The set of solutions are L.I.

Find the wronskian of $\{t, t^3\}$.

Ans: $W(t, t^3) = \begin{vmatrix} t & t^3 \\ 1 & 3t^2 \end{vmatrix}$

$$= 3t^3 - t^3$$
$$= 2t^3$$

$\therefore$ For $t = 0$, the set of solutions are L.D. otherwise linearly Independent.

Find the wronskian of $\{e^x, xe^x\}$

Ans: $W(e^x, xe^x) = \begin{vmatrix} e^x & xe^x \\ e^x & xe^x + e^x \end{vmatrix}$

$$= xe^{2x} + e^{2x} - xe^{2x}$$
$$= e^{2x} \neq 0, \forall x$$

$\therefore$ The set of solutions are L.I.

Find the wronskian of $\{x^3, |x|^3\}$ on $[-1, 1]$

Ans: for $x > 0$, $W(x^3, |x|^3)$

$$= \begin{vmatrix} x^3 & x^3 \\ 3x^2 & 3x^2 \end{vmatrix}$$

$$= 0.$$

For $x < 0$, $W(x^3, |x|^3)$

$$= \begin{vmatrix} x^3 & -x^3 \\ 3x^2 & -3x^2 \end{vmatrix}$$

$$= 0$$

For $x = 0$, $W(x^3, |x|^3)$

$$= \begin{vmatrix} 0 & 0 \\ 0 & 0 \end{vmatrix}$$

$$= 0.$$

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